# Supersonic Jet Inlet Using MacCormacks Method

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# Outline

# Motivation

- Jet engines cannot function in supersonic flows
  - Need carefully designed inlets to reduce air speed
  - Done through oblique shocks

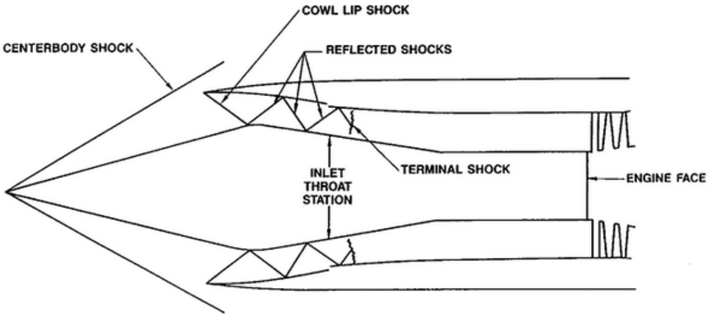



Figure: Example Inlet Design [1]

- A fast accurate numerical solver is desired

# Problem Statement

- Simplify geometry

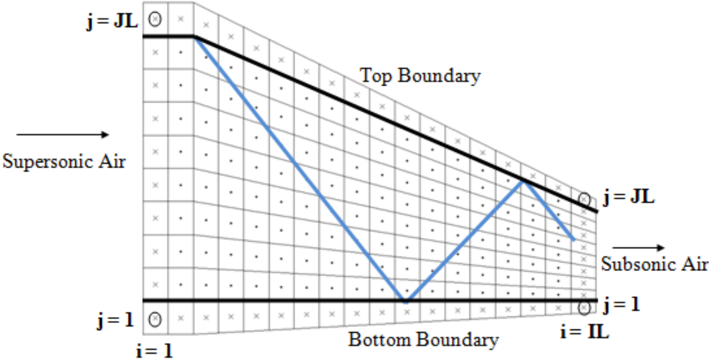
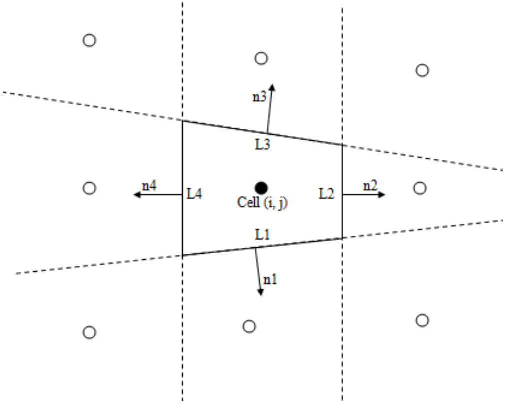



Figure: Simplified and Discretized Geometry

- Solve for the oblique shocks
- Design for specific flow parameters

# Cell Geometry

- Need geometric properties for each cell
  - Volume (Area in 2D)
  - Surface Areas (Edge Lengths in 2D)
  - Normals
- Schematic



# Exact Solution

- Normal Shock Relations

$$M_2{}^2 = \frac{(\gamma-1){M_1}^2+2}{2\gamma {M_1}^2-(\gamma-1)}$$

- $\theta, \beta, M$ relation

$$\tan(\theta) = 2\cot(\beta)\frac{{M_1}^2\sin^2\beta-1}{{M_1}^2(\gamma+\cot(2\beta))+2}$$

- Newton-Rhapson

$$\beta_{n+1} = \beta_n - \frac{f(\beta_n)}{f'(\beta_n)}$$

- Two step scheme
  - Predictor

$$U_{ij}^{\overline{n+1}} = U_{ij}^{\ n} - \frac{\Delta t}{A_{ij}}\left(\frac{\partial \dot{E}^{*}_{i+\frac{1}{2},j}}{\partial \xi}|L_{i\pm\frac{1}{2},j}| + \frac{\partial \dot{F}^{*}_{i,j+\frac{1}{2}}}{\partial \eta}|L_{i,j\pm\frac{1}{2}}|\right)$$

  - Corrector

$$U_{ij}^{\ n+1} = \frac{1}{2}\left[U_{ij}^{\ n} + U_{ij}^{\overline{n+1}} - \frac{\Delta t}{A_{ij}}\left(\frac{\partial \dot{E}^{**}_{i-\frac{1}{2},j}}{\partial \xi}|L_{i\pm\frac{1}{2},j}| + \frac{\partial \dot{F}^{**}_{i,j-\frac{1}{2}}}{\partial \eta}|L_{i,j\pm\frac{1}{2}}|\right)\right]$$

# Adding Dissipation

- Artificial Dissipation Terms (Subscript j is dropped for simplicity)

$$D_i = \epsilon(\vec{u}_i n_{4x} + \vec{v}_i n_{4y} + c_i)\frac{|P_i - 2P_{i-1} + P_{i-2}|}{P_i + 2P_{i-1} + P_{i-2}}$$

- Experimentally find optimal dissipation

# Convergence History

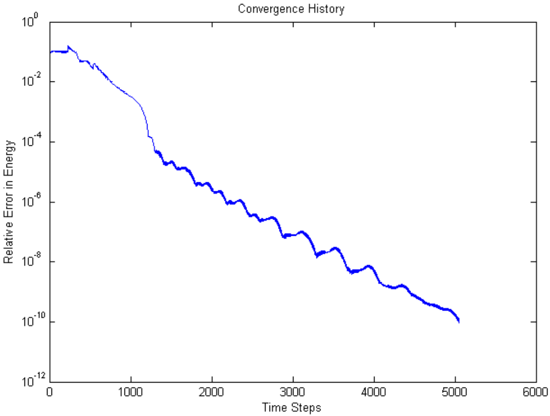



Figure: Convergence History for 4x Grid

# Optimal Dissipation

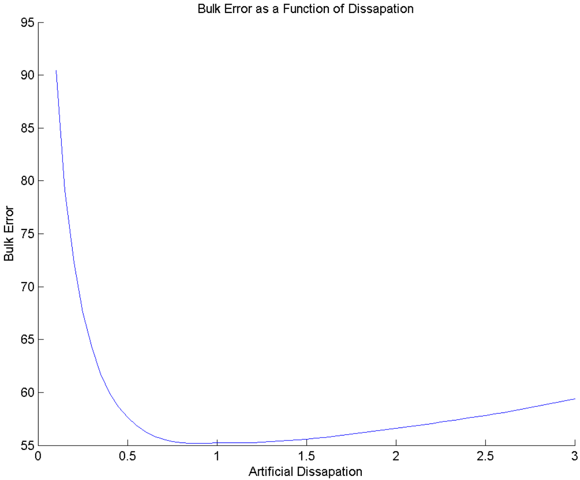
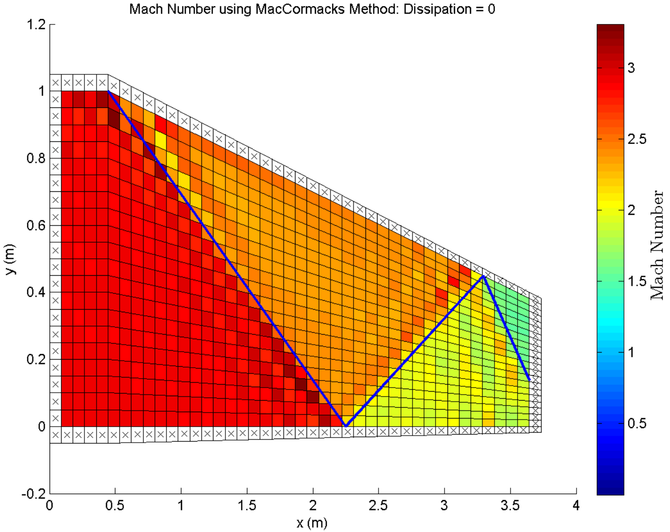



Figure: Optimal Dissipation Curve. Optimum at $\epsilon = 0.85$

# Zero Dissipation vs Optimal Dissipation



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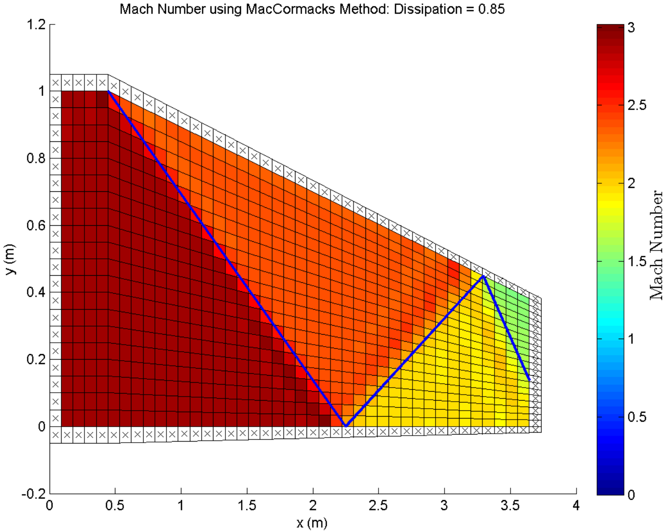
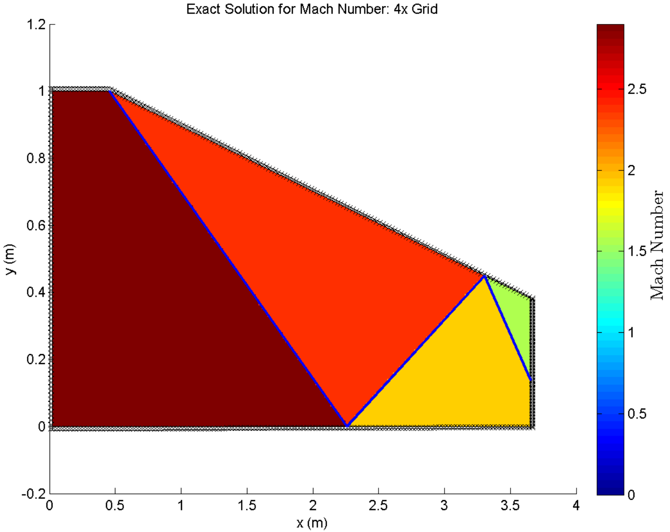


# Exact vs Numerical



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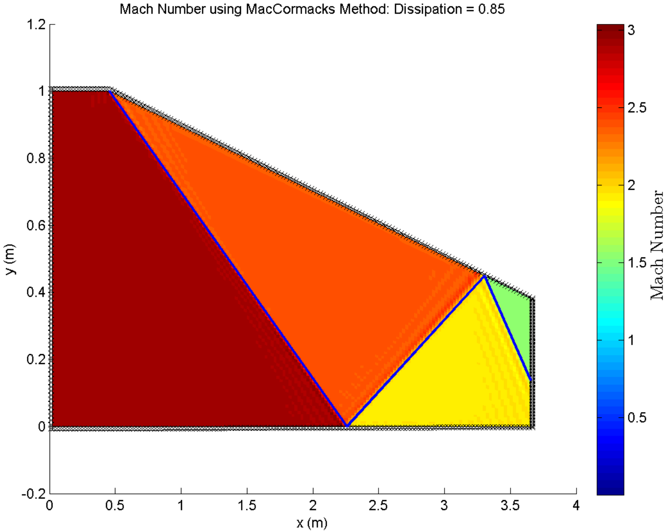

# Time Complexity

- Speed is important
- Sometimes a little accuracy is sacrifices for gains in speed
- Time complexities of algorithms are very important

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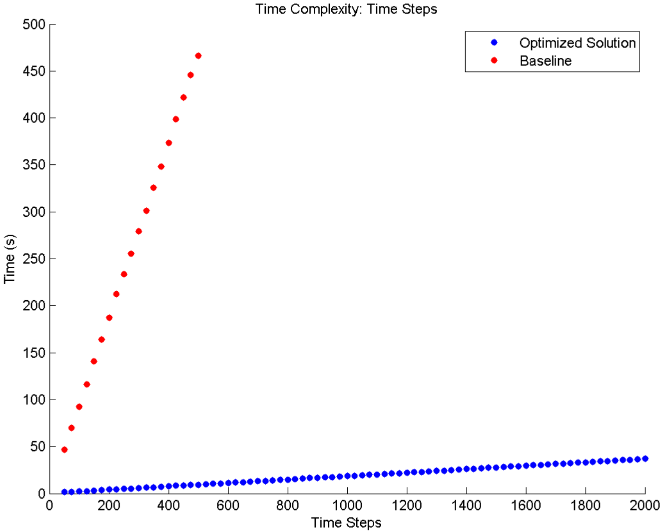

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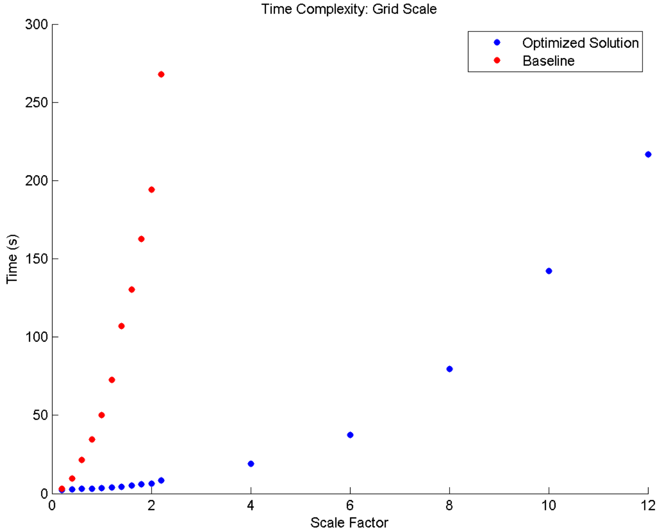

# Summary

- MacCormack Method with Artificial Dissipation has great accuracy with regards to the exact solution on all grids. As little as 1 percent average error per node.
- My implementation is fast and is an improvement of about 2 orders of magnitude in terms of runtime over the baseline
- Good numerical methods are needed to iterate and optimize designs

- Outlook
  - Apply to more general geometries
  - Make a good User Interface

# References

[ 1 ] "Bypass Air Systems." 456FIS. Web. Updated Feb. 10, 2014. "http://www.456fis.org/YF-12A_SR-71_ENGINE.html". Accessed Mar. 15, 2015